

# MECHANICAL WELLBORE STABILITY MODELLING IN ANISOTROPIC ROCKS

Ngurah Beni Setiawan and Robert Zimmerman, Department of Earth Science and Engineering, Imperial College London, UK

## Abstract

This work presents a unified approach through which the influence of the elastic and strength anisotropy on wellbore instability can be thoroughly examined. The stresses at the wellbore wall are first calculated using the Lekhnitskii-Amadei solution, which accounts for elastic anisotropy. Then, shear failure is treated by combining the Mogi-Coulomb criterion for intact rock, with the Jaeger plane of weakness concept. The developed model accounts for all three principal stresses in predicting the onset of shear failure. By having a more robust modelling approach, comprehensive examination of the instability propensity of the wellbore can be assessed, and drilling planning could be further improved, for instance, by way of optimising the mud weight design and obtaining a more favourable drilling direction.

Moreover, to investigate the post-drill stability of the wellbore, a semi-analytical solution based on graphical conformal mapping and complex variable methods is also proposed. The standard stress calculation method is limited to a wellbore with a circular cross-section, and hence can no longer be used. Yet, various purposes, such as post-drill open hole stability assessment, may still require information about the near-wellbore stress state after the wellbore has been damaged. The proposed method can be used to calculate the in-plane stress components in isotropic or anisotropic materials for arbitrarily-shaped holes. The method only requires the outline coordinates of the hole, the elastic moduli of the material, and the magnitude and direction of the far-field stresses. The solution is essentially closed-form, in the sense that it can be explicitly expressed in terms of the mapping coefficients and parameters that depend only on the elastic moduli of the materials. With such a degree of flexibility, the method is useful to study the effect of hole geometry on stress distribution around holes in isotropic or anisotropic materials.

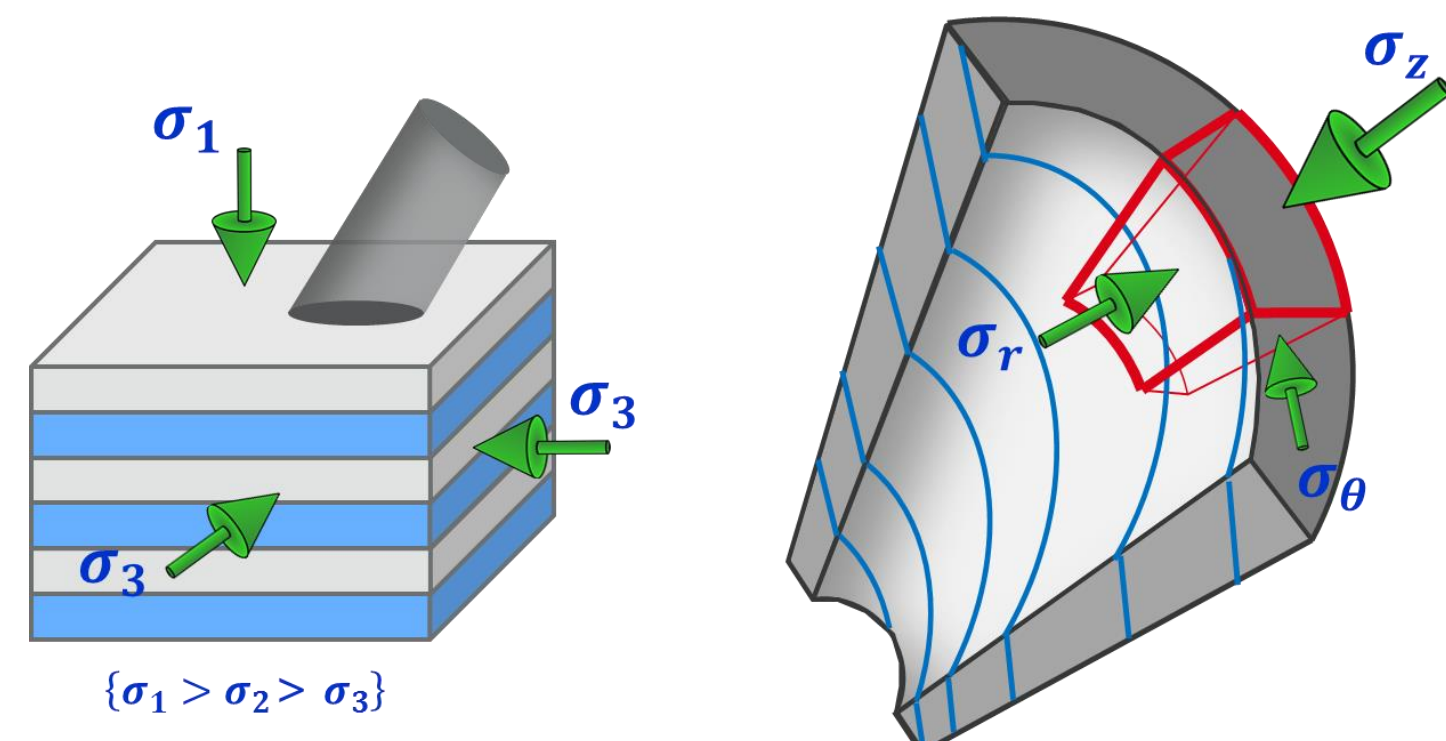
## Objectives

- To present a unified approach through which the influence of the elastic and strength anisotropy on wellbore instability can be thoroughly examined.
- To develop a method that allows the calculation of the stress around an arbitrary hole of given contour coordinate in isotropic or anisotropic media.

## Mechanical wellbore stability in anisotropic rocks

### The Lekhnitskii-Amadei solution

The Lekhnitskii-Amadei solution assumes an infinitely long borehole in an anisotropic medium, subjected to three far-field principal stresses. The degree of elastic anisotropy, in-situ stress, the wellbore internal pressure and wellbore trajectory will affect the stress around the wellbore.



The stress components in cartesian coordinate can be written as,

$$\begin{aligned}\sigma_{xx} &= \sigma_{x,o} + 2Re[\mu_1^2 \Phi_1'(z_1) + \mu_2^2 \Phi_2'(z_2) + \lambda_3 \mu_3^2 \Phi_3'(z_3)] \\ \sigma_{yy} &= \sigma_{y,o} + 2Re[\Phi_1'(z_1) + \Phi_2'(z_2) + \lambda_3 \Phi_3'(z_3)] \\ \tau_{xy} &= \tau_{xy,o} - 2Re[\mu_1 \Phi_1'(z_1) + \mu_2 \Phi_2'(z_2) + \lambda_3 \mu_3 \Phi_3'(z_3)] \\ \tau_{xz} &= \tau_{xz,o} + 2Re[\lambda_1 \mu_1 \Phi_1'(z_1) + \lambda_2 \mu_2 \Phi_2'(z_2) + \mu_3 \Phi_3'(z_3)] \\ \tau_{yz} &= \tau_{yz,o} - 2Re[\lambda_1 \Phi_1'(z_1) + \lambda_2 \Phi_2'(z_2) + \Phi_3'(z_3)]\end{aligned}$$

Where  $\mu_k$  is the root of a 6th-order polynomial from the Beltrami-Michell equation in which the material elastic property is embedded, and  $\Phi_k'(z_k)$  are the derivatives of analytic functions that depend on the internal hydrostatic pressure of the wellbore, the elastic properties of the anisotropic rocks and the radius of investigation.

### Effect of intermediate principal stress—Mogi-Coulomb failure criterion

To model the wellbore collapse under three-dimensional stress state, Mogi-Coulomb criterion is incorporated in the wellbore stability modelling. Proposed by Al-Ajmi and Zimmerman (2005), the Mogi-Coulomb criterion could predict true-triaxial behavior from traditional triaxial datasets even in the absence of true-triaxial data. The criterion is written as

$$\tau_{oct} = a + b\sigma_{m,2}$$

where  $\tau_{oct}$  is the critical octahedral stress and the mean effective stress  $\sigma_{m,2} = \frac{1}{2}(\sigma_1 + \sigma_3)$ .

### Effect of strength anisotropy—Jaeger's "Plane of Weakness" model

Some reservoir rocks, notably shales, are intrinsically anisotropic, and their physical properties, such as strength and deformability, will depend on the orientation of the rock with respect to the principal stresses. Jaeger (1960) postulated that for layered, transversely isotropic rocks, failure will occur either at a stress and at an angle given by the Coulomb criterion, or along a bedding plane. The Coulomb criterion for failure along the bedding plane is

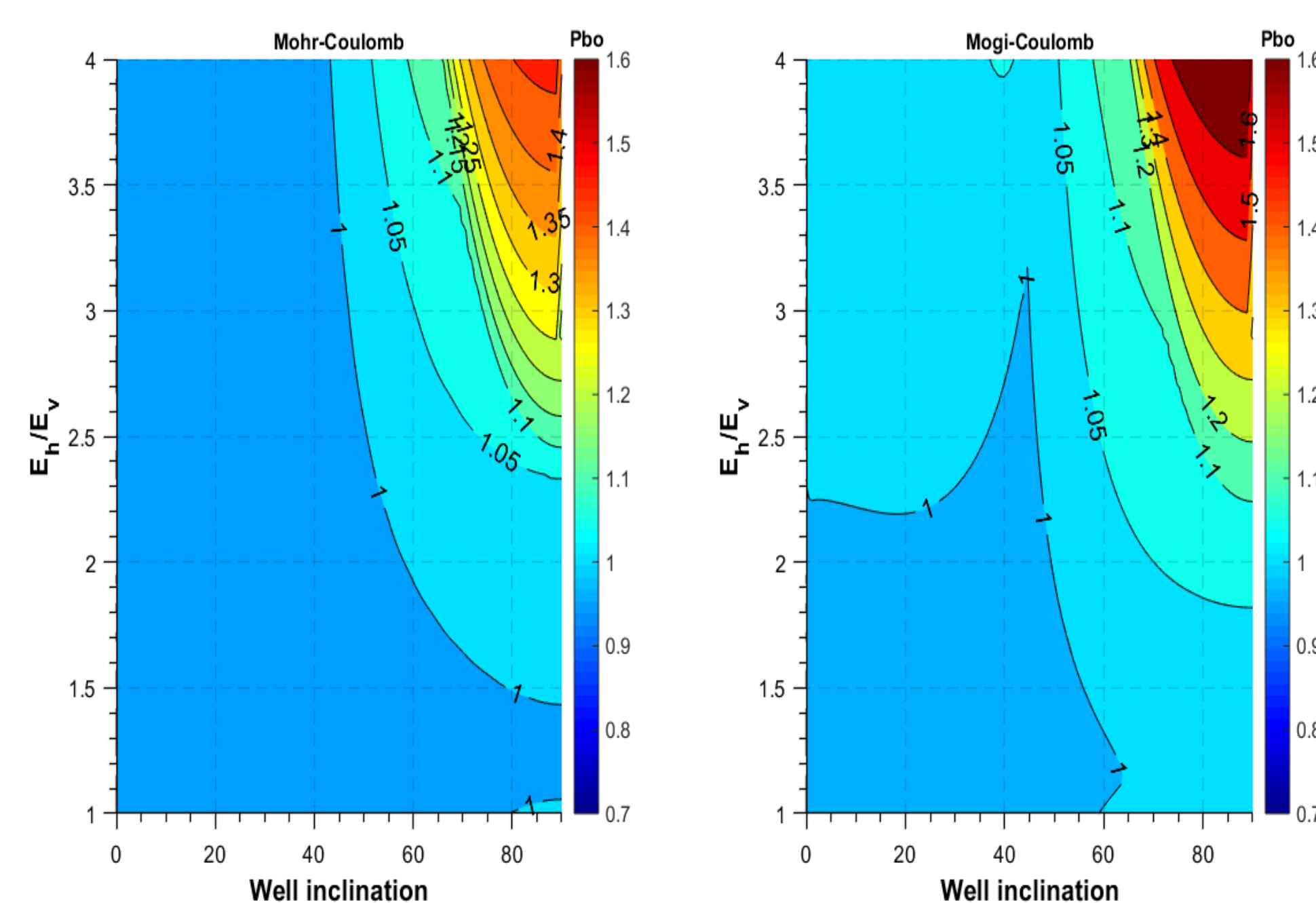
$$|\tau| = S_w + \mu_w \sigma_n$$

## References

- Al-Ajmi, A. M. and Zimmerman, R. W., 2005. Relation between the Mogi and the Coulomb failure criteria. *International Journal of Rock Mechanics and Mining Sciences*, 42:431-439.
- Jaeger, J. C. 1960. Shear Failure of Anisotropic Rocks. *Geological Magazine*, 97:65-72.
- Setiawan, N.B. and R.W. Zimmerman. 2018. Wellbore breakout prediction in transversely isotropic rocks using true-triaxial failure criteria. *Int. J. Rock Mech. Min. Sci.* 112:313-322.
- Setiawan, N.B. and R.W. Zimmerman. 2019a. A unified methodology for computing the stresses around an arbitrarily-shaped hole in isotropic or anisotropic materials. *Int. J. Solids and Structures*. (under review).
- Setiawan, N.B. and R.W. Zimmerman. 2019b. The Implications of using Anisotropic Elasticity and Fully-Triaxial Failure Criteria for Borehole Stability Analysis in Shales. *Paper ARMA 19-2178*.

## Drilling optimisation in anisotropic rocks

### Investigating the influence of elastic anisotropy in wellbore stability



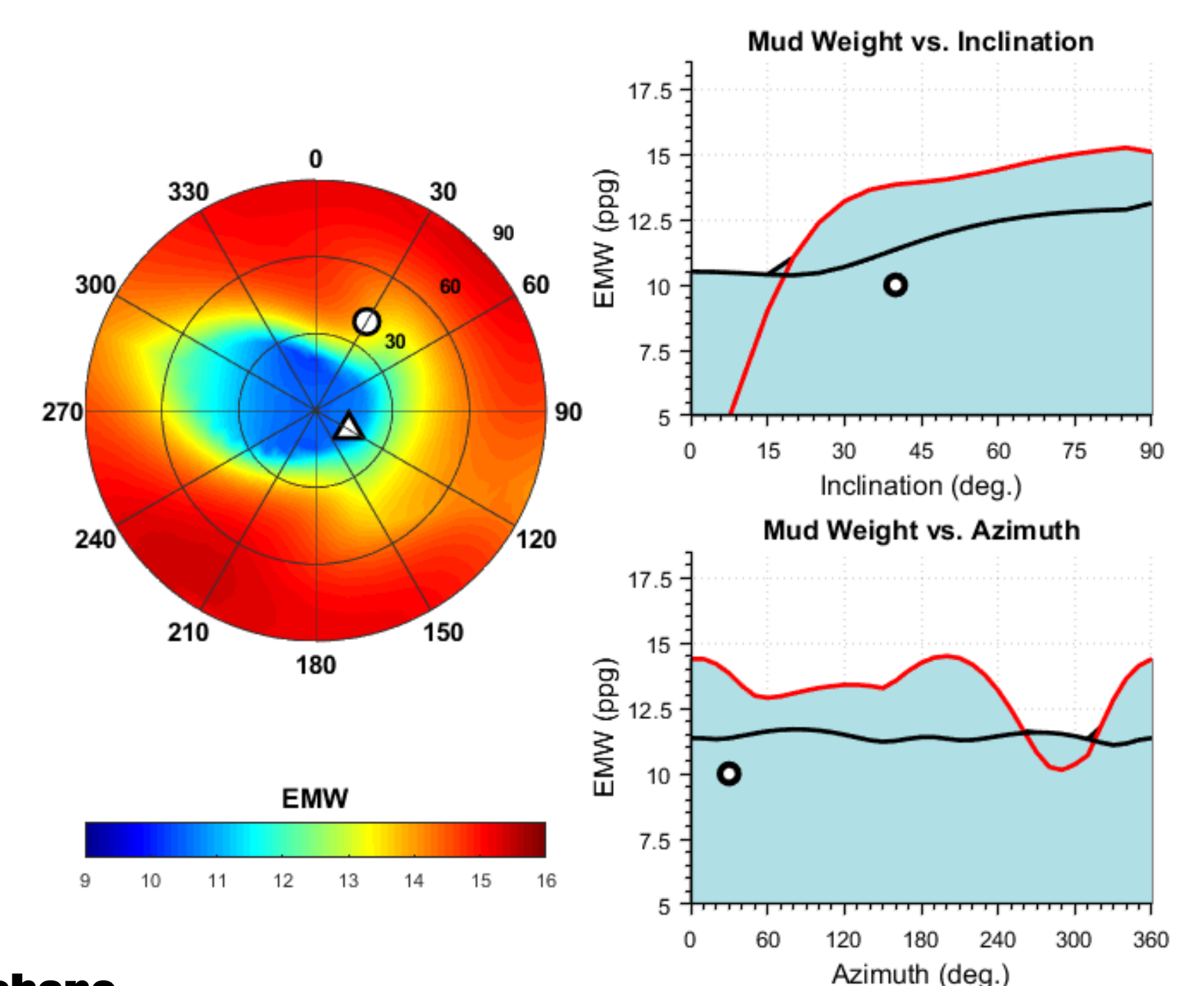
When Mohr-Coulomb criterion is considered, the effect of the elastic anisotropy seems to be negligible for a wellbore with deviation of less than 40°, for all elastic anisotropy ratios. Furthermore, if one considers a breakout pressure cut-off ratio of 5%, as compared with that of isotropic model, an anisotropy ratio  $E_v/E_h$  of 2.5 will only influence the breakout pressure when the wellbore deviation is higher than 70°.

However, according to the Mogi-Coulomb criterion, this 5% cut-off ratio is already observed even for a 60° wellbore deviation. Moreover, anisotropy will affect the breakout pressure for a nearly vertical wellbore with 5-10° deviation angle if  $E_v/E_h$  is 2.3 or greater. It is then reasonable to conclude that  $E_v/E_h = 2.3$  is the limiting value for which elastic anisotropy should not be neglected in the near-wellbore stress calculation. The dataset of this case is available in Setiawan and Zimmerman (2019b).

### Optimising mud weight and wellbore trajectory

By having a more robust modelling approach, comprehensive examination of the instability propensity of the wellbore can be assessed, and drilling planning could be further improved, for instance, by way of optimising the mud weight design and obtaining a more favourable drilling direction.

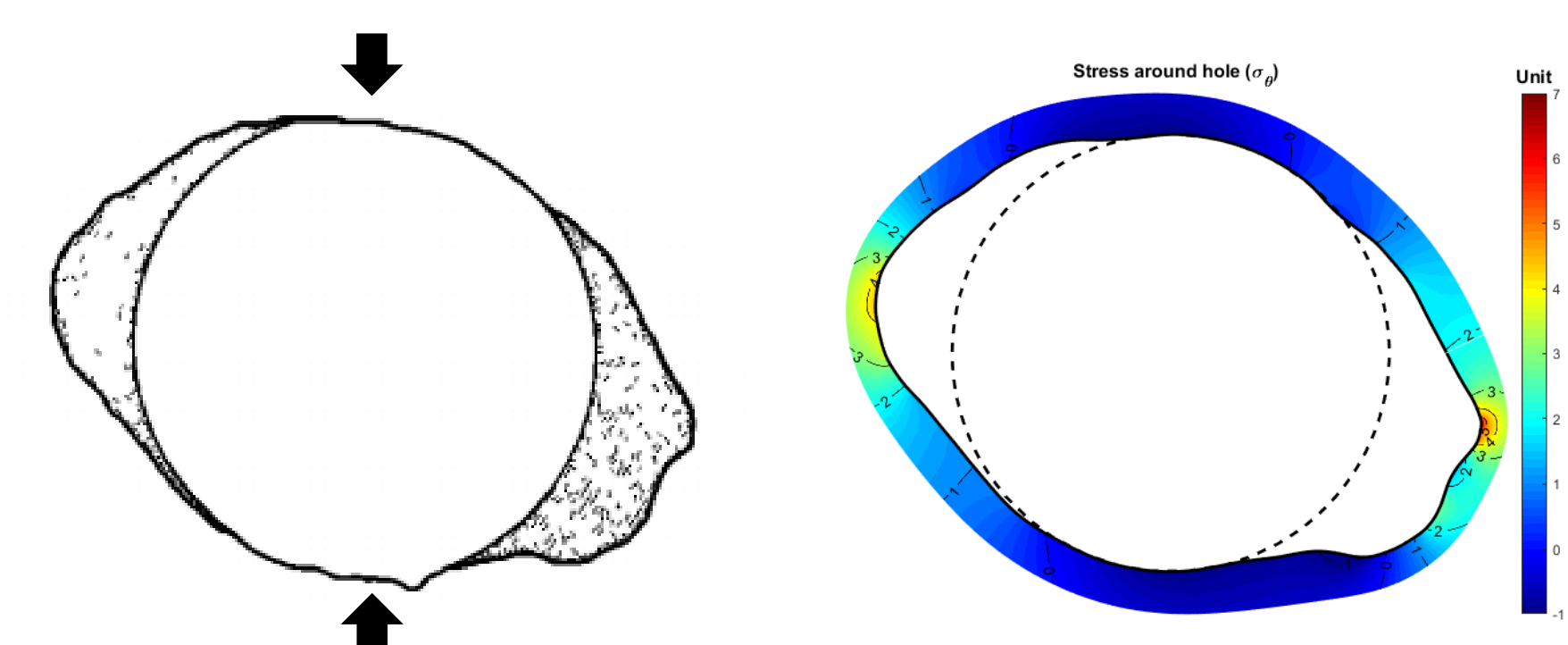
For instance, as shown in the figure, increasing the mud weight to at least 13.5 ppg is necessary to ensure the stability of the wellbore at this depth when both anisotropic elastic and strength properties are considered (Setiawan and Zimmerman, 2018).



### Stress around a non-symmetrical and irregular shape

A unified semi-analytical solution have also been developed to compute the stresses around a non-circular borehole, using Melentiev's conformal mapping algorithm. This methodology that can compute the stresses around an arbitrarily-shaped hole will perhaps find its greatest usefulness for irregularly-shaped holes.

One of the actual wellbore shapes reported by Zoback et al. (1985) has been digitised and the stress around the wellbore wall has been calculated using the proposed method to identify a region with high stress concentration (Setiawan and Zimmerman, 2019a).



## Conclusions

- The findings emphasise not only that considering intermediate stress is of great importance, but more importantly, neglecting elastic anisotropic characteristic would also affect the breakout pressure attributed to the reduction of the tangential stress and axial stress.
- New representation of stress potentials has been derived which depend on the conformal mapping constants obtained using Melentiev's iterative procedure. The method provides a unified approach to study the effect of hole geometry on stress distribution around isotropic or anisotropic materials, and should be particularly useful for irregularly-shaped holes that are not symmetrical or quasi-polygonal.